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Integrated Curriculum for AP Calculus and AP Physics

Based on the Standards of the BC Calculus and Physics C AP Examinations

The objective of this combined course is to cover the topics below by the end of March and spend the rest of the time on intensive preparation for the AP exams in May.

September

I. Functions, Graphs and Limits

Analysis of graphs. With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits)

• An intuitive understanding of the limiting process.
• Calculating limits using algebra.
• Estimating limits from graphs or tables of data.

Asymptotic and unbounded behavior

• Understanding asymptotes in terms of graphical behavior.
• Describing asymptotic behavior in terms of limits involving infinity.
• Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth and logarithmic growth).

Continuity as a property of functions

• An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
• Understanding continuity in terms of limits.
• Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).

Parametric, polar and vector functions

The analysis of planar curves includes those given in parametric form, polar form and vector form.

II. Derivatives

Concept of the derivative

• Derivative presented graphically, numerically and analytically.
• Derivative interpreted as an instantaneous rate of change.
• Derivative defined as the limit of the difference quotient.
• Relationship between differentiability and continuity.

Derivative at a point

• Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
• Tangent line to a curve at a point and local linear approximation.
• Instantaneous rate of change as the limit of average rate of change.
• Approximate rate of change from graphs and tables of values.

Derivative as a function

• Corresponding characteristics of graphs of f and f'.
• Relationship between the increasing and decreasing behavior of f and the sign of f'.
• The Mean Value Theorem and its geometric interpretation.

• Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives

• Corresponding characteristics of the graphs of f, f' and f''.

• Relationship between the concavity of f and the sign of f''.

• Points of inflection as places where concavity changes.

Applications of derivatives

• Analysis of curves, including the notions of monotonicity and concavity.

• Analysis of planar curves given in parametric form, polar form and vector form, including velocity and acceleration.

• Optimization, both absolute (global) and relative (local) extrema.

• Modeling rates of change, including related rates problems.

• Use of implicit differentiation to find the derivative of an inverse function.

• Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed and acceleration.

• Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.

Computation of derivatives

• Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric and inverse trigonometric functions.

• Derivative rules for sums, products and quotients of functions.

• Chain rule and implicit differentiation.

Derivatives of parametric, polar and vector functions.

NEWTONIAN MECHANICS

A. Kinematics (including vectors, vector algebra, components of vectors, coordinate systems, displacement, velocity, and acceleration)

1. Motion in one dimension

a) Students should understand the general relationships among position, velocity, and acceleration for the motion of a particle along a straight line, so that:

(1) Given a graph of one of the kinematic quantities, position, velocity, or acceleration, as a function of time, they can recognize in what time intervals the other two are positive, negative, or zero, and can identify or sketch a graph of each as a function of time.

(2) Given an expression for one of the kinematic quantities, position, velocity, or acceleration, as a function of time, they can determine the other two as a function of time, and find when these quantities are zero or achieve their maximum and minimum values.

b) Students should understand the special case of motion with constant acceleration, so they can:

(1) Write down expressions for velocity and position as functions of time, and identify or sketch graphs of these quantities.

(2) Solve problems involving one-dimensional motion with constant acceleration.

c) Students should know how to deal with situations in which acceleration is a specified function of velocity and time so they can write an appropriate differential equation and solve it for v by separation of variables, incorporating correctly a given initial value of v_0.

2. Motion in two dimensions, including projectile motion

a) Students should be able to add, subtract, and resolve displacement and velocity vectors, so they can:

(1) Determine components of a vector along two specified, mutually perpendicular axes.

(2) Determine the net displacement of a particle or the location of a particle relative to another.
(3) Determine the change in velocity of a particle or the velocity of one particle relative to another.
b) Students should understand the general motion of a particle in two dimensions so that, given functions $x(t)$ and $y(t)$ which describe this motion, they can determine the components, magnitude, and direction of the particle's velocity and acceleration as functions of time.
c) Students should understand the motion of projectiles in a uniform gravitational field, so they can:
(1) Write down expressions for the horizontal and vertical components of velocity and position as functions of time, and sketch or identify graphs of these components.
(2) Use these expressions in analyzing the motion of a projectile that is projected with an arbitrary initial velocity.

October
Applications of antidifferentiation
• Finding specific antiderivatives using initial conditions, including applications to motion along a line.
• Solving separable differential equations and using them in modeling (including the study of the equation $y' = ky$ and exponential growth).
Techniques of antidifferentiation
• Antiderivatives following directly from derivatives of basic functions.
• Antiderivatives by substitution of variables, parts, and simple partial fractions (nonrepeating linear factors only).
NEWTONIAN MECHANICS
B. Newton's laws of motion
1. Static equilibrium (first law)
Students should be able to analyze situations in which a particle remains at rest, or moves with constant velocity, under the influence of several forces.
2. Dynamics of a single particle (second law)
a) Students should understand the relation between the force that acts on an object and the resulting change in the object's velocity, so they can:
(1) Calculate, for an object moving in one dimension, the velocity change that results when a constant force F acts over a specified time interval.
(2) Calculate, for an object moving in one dimension, the velocity change that results when a force F(t) acts over a specified time interval.
(3) Determine, for an object moving in a plane whose velocity vector undergoes a specified change over a specified time interval, the average force that acted on the object.
b) Students should understand how Newton's Second Law applies to an object subject to forces such as gravity, the pull of strings, or contact forces, so they can:
(1) Draw a well-labeled, free-body diagram showing all real forces that act on the object.
(2) Write down the vector equation that results from applying Newton's Second Law to the object, and take components of this equation along appropriate axes.
c) Students should be able to analyze situations in which an object moves with specified acceleration under the influence of one or more forces so they can determine the magnitude and direction of the net force, or of one of the forces that makes up the net force, such as motion up or down with constant acceleration.

d) Students should understand the significance of the coefficient of friction, so they can:
(1) Write down the relationship between the normal and frictional forces on a surface.
(2) Analyze situations in which an object moves along a rough inclined plane or horizontal surface.
(3) Analyze under what circumstances an object will start to slip, or to calculate the magnitude of the force of static friction.
e) Students should understand the effect of drag forces on the motion of an object, so they can:
(1) Find the terminal velocity of an object moving vertically under the influence of a retarding force dependent on velocity.
(2) Describe qualitatively, with the aid of graphs, the acceleration, velocity, and displacement of such a particle when it is released from rest or is projected vertically with specified initial velocity.
(3) Use Newton's Second Law to write a differential equation for the velocity of the object as a function of time.
(4) Use the method of separation of variables to derive the equation for the velocity as a function of time from the differential equation that follows from Newton's Second Law.
(5) Derive an expression for the acceleration as a function of time for an object falling under the influence of drag forces.
3. Systems of two or more objects (third law)
a) Students should understand Newton's Third Law so that, for a given system, they can identify the force pairs and the objects on which they act, and state the magnitude and direction of each force.
b) Students should be able to apply Newton's Third Law in analyzing the force of contact between two objects that accelerate together along a horizontal or vertical line, or between two surfaces that slide across one another.
c) Students should know that the tension is constant in a light string that passes over a massless pulley and should be able to use this fact in analyzing the motion of a system of two objects joined by a string.
d) Students should be able to solve problems in which application of Newton's laws leads to two or three simultaneous linear equations involving unknown forces or accelerations.

November

III. Integrals

Interpretations and properties of definite integrals

• Definite integral as a limit of Riemann sums.
• Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval.
• Basic properties of definite integrals (examples include additivity and linearity).

Applications of integrals. Appropriate integrals are used in a variety of applications to model physical, biological or economic situations. The emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. Specific applications include finding the area of a region (including a region bounded by polar curves), the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, the length of a curve (including a curve given in parametric form), and accumulated change from a rate of change.

Fundamental Theorem of Calculus

• Use of the Fundamental Theorem to evaluate definite integrals.

• Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.
• Improper integrals (as limits of definite integrals).
• L'Hospital's Rule, including its use in determining limits and convergence of improper integrals and series.
• Solving logistic differential equations and using them in modeling.
• Numerical solution of differential equations using Euler's method.

Numerical approximations to definite integrals.

Use of Riemann sums
(using left, right and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically and by tables of values.

NEWTONIAN MECHANICS (CONTINUED)

C. Work, energy, power

1. Work and the work-energy theorem

a) Students should understand the definition of work, including when it is positive, negative, or zero, so they can:

(1) Calculate the work done by a specified constant force on an object that undergoes a specified displacement.

(2) Relate the work done by a force to the area under a graph of force as a function of position, and calculate this work in the case where the force is a linear function of position.

(3) Use integration to calculate the work performed by a force F(x) on an object that undergoes a specified displacement in one dimension.

(4) Use the scalar product operation to calculate the work performed by a specified constant force F on an object that undergoes a displacement in a plane.

b) Students should understand and be able to apply the work-energy theorem, so they can:

(1) Calculate the change in kinetic energy or speed that results from performing a specified amount of work on an object.

(2) Calculate the work performed by the net force, or by each of the forces that make up the net force, on an object that undergoes a specified change in speed or kinetic energy.

(3) Apply the theorem to determine the change in an object's kinetic energy and speed that results from the application of specified forces, or to determine the force that is required in order to bring an object to rest in a specified distance.

2. Forces and potential energy

a) Students should understand the concept of a conservative force, so they can:

(1) State alternative definitions of "conservative force" and explain why these definitions are equivalent.

(2) Describe examples of conservative forces and non-conservative forces.

b) Students should understand the concept of potential energy, so they can:

(1) State the general relation between force and potential energy, and explain why potential energy can be associated only with conservative forces.

(2) Calculate a potential energy function associated with a specified one-dimensional force F(x).

(3) Calculate the magnitude and direction of a one-dimensional force when given the potential energy function U(x) for the force.

(4) Write an expression for the force exerted by an ideal spring and for the potential energy of a stretched or compressed spring.

(5) Calculate the potential energy of one or more objects in a uniform gravitational field.

3. Conservation of energy
a) Students should understand the concepts of mechanical energy and of total energy, so they can:
(1) State and apply the relation between the work performed on an object by non-conservative forces and the change in an object's mechanical energy.
(2) Describe and identify situations in which mechanical energy is converted to other forms of energy.
(3) Analyze situations in which an object's mechanical energy is changed by friction or by a specified externally applied force.
b) Students should understand conservation of energy, so they can:
(1) Identify situations in which mechanical energy is or is not conserved.
(2) Apply conservation of energy in analyzing the motion of systems of connected objects, such as an Atwood's machine.
(3) Apply conservation of energy in analyzing the motion of objects that move under the influence of springs.
(4) Apply conservation of energy in analyzing the motion of objects that move under the influence of other non-constant one-dimensional forces.
c) Students should be able to recognize and solve problems that call for application both of conservation of energy and Newton's Laws.

December
4. Power
Students should understand the definition of power, so they can:
a) Calculate the power required to maintain the motion of an object with constant acceleration (e.g., to move an object along a level surface, to raise an object at a constant rate, or to overcome friction for an object that is moving at a constant speed).
b) Calculate the work performed by a force that supplies constant power, or the average power supplied by a force that performs a specified amount of work.
D. Systems of particles, linear momentum
1. Center of mass
a) Students should understand the technique for finding center of mass, so they can:
(1) Identify by inspection the center of mass of a symmetrical object.
(2) Locate the center of mass of a system consisting of two such objects.
(3) Use integration to find the center of mass of a thin rod of non-uniform density
b) Students should be able to understand and apply the relation between center-of-mass velocity and linear momentum, and between center-of-mass acceleration and net external force for a system of particles.
c) Students should be able to define center of gravity and to use this concept to express the gravitational potential energy of a rigid object in terms of the position of its center of mass.
2. Impulse and momentum
Students should understand impulse and linear momentum, so they can:
a) Relate mass, velocity, and linear momentum for a moving object, and calculate the total linear momentum of a system of objects.
b) Relate impulse to the change in linear momentum and the average force acting on an object.
c) State and apply the relations between linear momentum and center-of-mass motion for a system of particles.

d) Calculate the area under a force versus time graph and relate it to the change in momentum of an object.
e) Calculate the change in momentum of an object given a function F(t) for the net force acting on the object.
3. Conservation of linear momentum, collisions
a) Students should understand linear momentum conservation, so they can:
(1) Explain how linear momentum conservation follows as a consequence of Newton's Third Law for an isolated system.
(2) Identify situations in which linear momentum, or a component of the linear momentum vector, is conserved.
(3) Apply linear momentum conservation to one-dimensional elastic and inelastic collisions and two-dimensional completely inelastic collisions.
(4) Apply linear momentum conservation to two-dimensional elastic and inelastic collisions.
(5) Analyze situations in which two or more objects are pushed apart by a spring or other agency, and calculate how much energy is released in such a process.
b) Students should understand frames of reference, so they can:
(1) Analyze the uniform motion of an object relative to a moving medium such as a flowing stream.
(2) Analyze the motion of particles relative to a frame of reference that is accelerating horizontally or vertically at a uniform rate.

January
E. Circular motion and rotation
1. Uniform circular motion
Students should understand the uniform circular motion of a particle, so they can:
a) Relate the radius of the circle and the speed or rate of revolution of the particle to the magnitude of the centripetal acceleration.
b) Describe the direction of the particle's velocity and acceleration at any instant during the motion.
c) Determine the components of the velocity and acceleration vectors at any instant, and sketch or identify graphs of these quantities.
d) Analyze situations in which an object moves with specified acceleration under the influence of one or more forces so they can determine the magnitude and direction of the net force, or of one of the forces that makes up the net force, in situations such as the following:
(1) Motion in a horizontal circle (e.g., mass on a rotating merry-go-round, or car rounding a banked curve).
(2) Motion in a vertical circle (e.g., mass swinging on the end of a string, cart rolling down a curved track, rider on a Ferris wheel).
2. Torque and rotational statics
a) Students should understand the concept of torque, so they can:
(1) Calculate the magnitude and direction of the torque associated with a given force.
(2) Calculate the torque on a rigid object due to gravity.
b) Students should be able to analyze problems in statics, so they can:
(1) State the conditions for translational and rotational equilibrium of a rigid object.
(2) Apply these conditions in analyzing the equilibrium of a rigid object under the combined influence of a number of coplanar forces applied at different locations.

c) Students should develop a qualitative understanding of rotational inertia, so they can:
(1) Determine by inspection which of a set of symmetrical objects of equal mass has the greatest rotational inertia.
(2) Determine by what factor an object's rotational inertia changes if all its dimensions are increased by the same factor.
d) Students should develop skill in computing rotational inertia so they can find the rotational inertia of:
(1) A collection of point masses lying in a plane about an axis perpendicular to the plane.
(2) A thin rod of uniform density, about an arbitrary axis perpendicular to the rod.
(3) A thin cylindrical shell about its axis, or an object that may be viewed as being made up of coaxial shells.
e) Students should be able to state and apply the parallel-axis theorem.
3. Rotational kinematics and dynamics
a) Students should understand the analogy between translational and rotational kinematics so they can write and apply relations among the angular acceleration, angular velocity, and angular displacement of an object that rotates about a fixed axis with constant angular acceleration.
b) Students should be able to use the right-hand rule to associate an angular velocity vector with a rotating object.
c) Students should understand the dynamics of fixed-axis rotation, so they can:
(1) Describe in detail the analogy between fixed-axis rotation and straight-line translation.
(2) Determine the angular acceleration with which a rigid object is accelerated about a fixed axis when subjected to a specified external torque or force.
(3) Determine the radial and tangential acceleration of a point on a rigid object.
(4) Apply conservation of energy to problems of fixed-axis rotation.
(5) Analyze problems involving strings and massive pulleys.
d) Students should understand the motion of a rigid object along a surface, so they can:
(1) Write down, justify, and apply the relation between linear and angular velocity, or between linear and angular acceleration, for an object of circular cross-section that rolls without slipping along a fixed plane, and determine the velocity and acceleration of an arbitrary point on such an object.
(2) Apply the equations of translational and rotational motion simultaneously in analyzing rolling with slipping.
(3) Calculate the total kinetic energy of an object that is undergoing both translational and rotational motion, and apply energy conservation in analyzing such motion.
4. Angular momentum and its conservation
a) Students should be able to use the vector product and the right-hand rule, so they can:
(1) Calculate the torque of a specified force about an arbitrary origin.
(2) Calculate the angular momentum vector for a moving particle.
(3) Calculate the angular momentum vector for a rotating rigid object in simple cases where this vector lies parallel to the angular velocity vector.
b) Students should understand angular momentum conservation, so they can:
(1) Recognize the conditions under which the law of conservation is applicable and relate this law to one- and two-particle systems such as satellite orbits.
(2) State the relation between net external torque and angular momentum, and identify situations in which angular momentum is conserved.

(3) Analyze problems in which the moment of inertia of an object is changed as it rotates freely about a fixed axis.
(4) Analyze a collision between a moving particle and a rigid object that can rotate about a fixed axis or about its center of mass.

February
F. Oscillations and Gravitation
1. Simple harmonic motion (dynamics and energy relationships)
Students should understand simple harmonic motion, so they can:
a) Sketch or identify a graph of displacement as a function of time, and determine from such a graph the amplitude, period, and frequency of the motion.
b) Write down an appropriate expression for displacement of the form $A\sin\omega t$ or $A\cos\omega t$ to describe the motion.
c) Find an expression for velocity as a function of time.
d) State the relations between acceleration, velocity, and displacement, and identify points in the motion where these quantities are zero or achieve their greatest positive and negative values.
e) State and apply the relation between frequency and period.
f) Recognize that a system that obeys a differential equation of the form $\frac{d^2x}{dt^2} = -\omega^2 x$ must execute simple harmonic motion, and determine the frequency and period of such motion.
g) State how the total energy of an oscillating system depends on the amplitude of the motion, sketch or identify a graph of kinetic or potential energy as a function of time, and identify points in the motion where this energy is all potential or all kinetic.
h) Calculate the kinetic and potential energies of an oscillating system as functions of time, sketch or identify graphs of these functions, and prove that the sum of kinetic and potential energy is constant.
i) Calculate the maximum displacement or velocity of a particle that moves in simple harmonic motion with specified initial position and velocity.
j) Develop a qualitative understanding of resonance so they can identify situations in which a system will resonate in response to a sinusoidal external force.
2. Mass on a spring
Students should be able to apply their knowledge of simple harmonic motion to the case of a mass on a spring, so they can:
a) Derive the expression for the period of oscillation of a mass on a spring.
b) Apply the expression for the period of oscillation of a mass on a spring.
c) Analyze problems in which a mass hangs from a spring and oscillates vertically.
d) Analyze problems in which a mass attached to a spring oscillates horizontally.
e) Determine the period of oscillation for systems involving series or parallel combinations of identical springs, or springs of differing lengths.
3. Pendulum and other oscillations
Students should be able to apply their knowledge of simple harmonic motion to the case of a pendulum, so they can:
a) Derive the expression for the period of a simple pendulum.
b) Apply the expression for the period of a simple pendulum.
c) State what approximation must be made in deriving the period of a simple pendulum.
d) Analyze the motion of a torsional pendulum or physical pendulum in order to determine the period of small oscillations.

4. Newton's law of gravity
Students should know Newton's Law of Universal Gravitation, so they can:
a) Determine the force that one spherically symmetrical mass exerts on another.
b) Determine the strength of the gravitational field at a specified point outside a spherically symmetrical mass.
c) Describe the gravitational force inside and outside a uniform sphere, and calculate how the field at the surface depends on the radius and density of the sphere.
5. Orbits of planets and satellites
Students should understand the motion of an object in orbit under the influence of gravitational forces, so they can:
a) For a circular orbit:
(1) Recognize that the motion does not depend on the object's mass; describe qualitatively how the velocity, period of revolution, and centripetal acceleration depend upon the radius of the orbit; and derive expressions for the velocity and period of revolution in such an orbit.
(2) Derive Kepler's Third Law for the case of circular orbits.
(3) Derive and apply the relations among kinetic energy, potential energy, and total energy for such an orbit.
b) For a general orbit:
(1) State Kepler's three laws of planetary motion and use them to describe in qualitative terms the motion of an object in an elliptical orbit.
(2) Apply conservation of angular momentum to determine the velocity and radial distance at any point in the orbit.
(3) Apply angular momentum conservation and energy conservation to relate the speeds of an object at the two extremes of an elliptical orbit.
(4) Apply energy conservation in analyzing the motion of an object that is projected straight up from a planet's surface or that is projected directly toward the planet from far above the surface.

March
IV. Polynomial Approximations and Series
Concept of series.
A series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequence of partial sums. Technology can be used to explore convergence and divergence.
Series of constants
• Motivating examples, including decimal expansion.
• Geometric series with applications.
• The harmonic series.
• Alternating series with error bound.
• Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series.
• The ratio test for convergence and divergence.
• Comparing series to test for convergence or divergence.
Taylor series
• Taylor polynomial approximation with graphical demonstration of convergence (for example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve).
• Maclaurin series and the general Taylor series centered at $x = a$.
• Maclaurin series for the functions e^x , $\sin x$, $\cos x$, and $\frac{1}{1-x}$.

• Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation and the formation of new series from known series.
• Functions defined by power series.
• Radius and interval of convergence of power series.
• Lagrange error bound for Taylor polynomials.

ELECTRICITY AND MAGNETISM (Time permitting)

A. Electrostatics

1. Charge and Coulomb's Law

a) Students should understand the concept of electric charge, so they can:
(1) Describe the types of charge and the attraction and repulsion of charges.
(2) Describe polarization and induced charges.
b) Students should understand Coulomb's Law and the principle of superposition, so they can:
(1) Calculate the magnitude and direction of the force on a positive or negative charge due to other specified point charges.
(2) Analyze the motion of a particle of specified charge and mass under the influence of an electrostatic force.

2. Electric field and electric potential (including point charges)

a) Students should understand the concept of electric field, so they can:
(1) Define it in terms of the force on a test charge.
(2) Describe and calculate the electric field of a single point charge.
(3) Calculate the magnitude and direction of the electric field produced by two or more point charges.
(4) Calculate the magnitude and direction of the force on a positive or negative charge placed in a specified field.
(5) Interpret an electric field diagram.
(6) Analyze the motion of a particle of specified charge and mass in a uniform electric field.
b) Students should understand the concept of electric potential, so they can:
(1) Determine the electric potential in the vicinity of one or more point charges.
(2) Calculate the electrical work done on a charge or use conservation of energy to determine the speed of a charge that moves through a specified potential difference.
(3) Determine the direction and approximate magnitude of the electric field at various positions given a sketch of equipotentials.
(4) Calculate the potential difference between two points in a uniform electric field, and state which point is at the higher potential.
(5) Calculate how much work is required to move a test charge from one location to another in the field of fixed point charges.
(6) Calculate the electrostatic potential energy of a system of two or more point charges, and calculate how much work is required to establish the charge system.
(7) Use integration to determine electric potential difference between two points on a line, given electric field strength as a function of position along that line.
(8) State the general relationship between field and potential, and define and apply the concept of a conservative electric field.
3. Gauss's law